Quasi-classical physics and T-linear resistivity in both strongly correlated and ordinary metals

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We show that near a quantum critical point generating quantum criticality of strongly correlated metals where the density of electron states diverges, the quasi-classical physics remains applicable to the description of the resistivity ρ of strongly correlated metals due to the presence of a transverse zero-sound collective mode, reminiscent of the phonon mode in solids. We demonstrate that at T, being in excess of an extremely low Debye temperature T_D, the resistivity $\rho(T)$ changes linearly with T, since the mechanism, forming the T dependence of $\rho(T)$, is the same as the electron-phonon mechanism that prevails at high temperatures in ordinary metals. Thus, electron-phonon scattering leads to near material-independence of the lifetime τ of quasiparticles that is expressed as the ratio of the Planck constant $\hbar$ to the Boltzmann constant k_B, $T\tau \sim \hbar/k_B$. We find that at $T < T_D$ there exists a different mechanism, maintaining the T-linear dependence of $\rho(T)$, and making the constancy of τ fail in spite of the presence of T-linear dependence. Our results are in good agreement with exciting experimental observations.

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Discoveries of surprising universality in the properties of both strongly correlated metals and ordinary ones provide unique opportunities for checking and expanding our understanding of quantum criticality in strongly correlated compounds. When exploring at different temperatures T a linear in temperature resistivity of these utterly different metals, an universality of their fundamental physical properties has been revealed [1]. On one hand, at low T the linear T-resistivity

$$\rho(T) = \rho_0 + AT, \tag{1}$$

observed in many strongly correlated compounds such as high-temperature superconductors and heavy-fermion metals located near their quantum critical points and therefore exhibiting quantum criticality. Here ρ_0 is the residual resistivity and A is a T-independent coefficient. On the other hand, at room temperatures the same behavior is exhibited by conventional metals such as Al, Ag or Cu. In case of a simple metal with a single Fermi surface pocket the resistivity reads $e^2 n\rho = p_F/(\tau v_F)$ [2], where e is the electronic charge, τ is the lifetime, n is the carrier concentration, and p_F and v_F are the Fermi momentum and the Fermi velocity, correspondingly. Writing the lifetime τ (or inverse scattering rate) of quasiparticles in the form [3, 4]

$$\frac{\hbar}{\tau} \simeq a_1 + \frac{k_B T}{a_2}, \tag{2}$$

we obtain

$$a_2 \frac{e^2 n\hbar}{p_F k_B}\frac{\partial\rho}{\partial T} = \frac{1}{v_F}, \tag{3}$$

where $\hbar$ is Planck's constant, k_B is Boltzmanns constant, a_1 and a_2 are T-independent parameters. A challenging point is that experimental facts corroborate Eq. (3) in case of both strongly correlated metals and ordinary ones provided that these demonstrate the linear T-dependence of their resistivity [1]. Moreover, the analysis of data available in literature for the most various compounds with the linear dependence of $\rho(T)$ shows: The coefficient a_2 is always close to unit, $0.7 \leq a_2 \leq 2.7$, notwithstanding huge distinction in the absolute value of ρ, T and Fermi velocities v_F, varying by two orders of magnitude [1]. As a result, it follows from Eq. (2) that the T-linear scattering rate is of universal form, $1/(\tau T) \sim k_B/\hbar$, regardless of different systems displaying the T-linear dependence. Indeed, this dependence is demonstrated by ordinary metals at temperatures higher than the Debye temperature, $T \geq T_D$, with an electron - phonon mechanism and by strongly correlated metals which are assumed to be fundamentally different from the ordinary ones, in which the linear dependence at their quantum criticality and temperatures of a few Kelvin is assumed to come from excitations of electronic origin rather than from phonons [1].

In this letter, we show that the quasi-classical physics is still applicable to describe the T-linear dependence of resistivity of strongly correlated metals at their quantum criticality since flat bands, forming the quantum criticality, generate transverse zero-sound mode with the Debye temperature T_D strongly depending on temperature. At

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$T \geq T_D$ the mechanism of the T-linear dependence is the same both in ordinary metals and strongly correlated ones, and is represented by electron-phonon scattering. Therefore, it is electron-phonon scattering at $T \geq T_D$ that leads to the near material-independence of the lifetime τ that is expressed as $1/(\tau T) \sim k_B/\hbar$. We show there can be another mechanism supporting the T-linear dependence even at $T < T_D$, which makes fail the constancy of τ.

To develop explanations of constancy of T-linear scattering rate $1/(\tau T)$, it is necessary to recall the nature and consequences of flattening of single-particle excitation spectra $\varepsilon(\mathbf{p})$ ("flat bands") in strongly correlated Fermi systems [5–8] (for recent reviews, see [9–11]). At $T = 0$, the ground state of a system with a flat band is degenerate, and the occupation numbers $n_0(\mathbf{p})$ of single-particle states belonging to the flat band are continuous functions of momentum $\mathbf{p}$, in contrast to discrete standard Landau Fermi liquid (LFL) values 0 and 1, as it seen from Fig. 1. Such behavior of $n_0(\mathbf{p})$ leads to a temperature-independent entropy term

$$S_0 = -\sum_{\mathbf{p}}[n_0(\mathbf{p})\ln n_0(\mathbf{p}) + (1-n_0(\mathbf{p}))\ln(1-n_0(\mathbf{p}))]. \tag{4}$$

Unlike the corresponding LFL entropy, which vanishes linearly as $T \to 0$, the term S_0 produces the non-Fermi liquid (NFL) behavior that includes T-independent thermal expansion coefficient [4, 9, 12, 13]. That T-independent behavior is observed in measurements on $CeCoIn_5$ [14–16] and $YbRh_2(Si_{0.95}Ge_{0.05})_2$ [17], while very recent measurements on $Sr_3Ru_2O_7$ indicate the same behavior [18, 19]. In the theory of fermion condensation (FC), the degeneracy of the NFL ground state is removed at any finite temperature, with the flat band acquiring a small dispersion [7, 9]

$$\varepsilon(\mathbf{p}) = \mu + T\ln\frac{1-n_0(\mathbf{p})}{n_0(\mathbf{p})} \tag{5}$$

proportional to T with μ being the chemical potential. The occupation numbers n_0 of FC remain unchanged at relatively low temperatures and, accordingly, so does the entropy S_0. Due to the fundamental difference between the FC single-particle spectrum and that of the remainder of the Fermi liquid, a system having FC is, in fact, a two-component system. The range L of momentum space adjacent to the Fermi surface where FC resides is given by $L \simeq (p_f - p_i)$, as seen from Fig. 1.

In strongly correlated metals at high temperatures, a light electronic band coexists with f or d-electron narrow bands placed below the Fermi surface. At lower temperatures when the quantum criticality is formed, a hybridization between this light band and f or d-electron bands results in its splitting into new flat bands, while some of the bands remain light representing LFL states [20]. A flat band can also be formed by a van Hove singularity (vHs) [21–28]. We assume that at least one of these flat bands crosses the Fermi level and represents the FC subsystem shown in Fig. 1. Remarkably, the FC subsystem

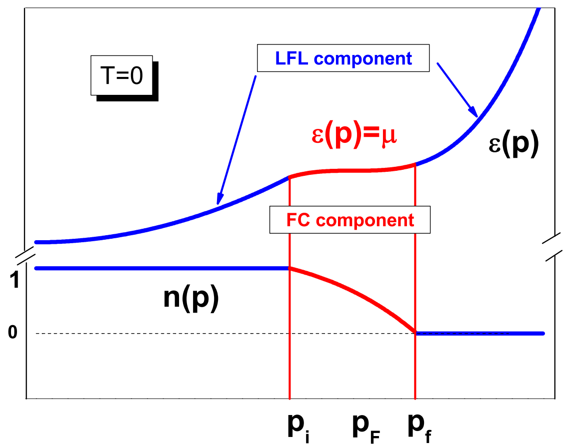

FIG. 1: (Color on line) Schematic plot of two-component electron liquid at $T = 0$ with FC. Due to the presence of FC, the system is separated into two components: The first component is a normal liquid with the quasiparticle distribution function $n_0(p < p_i) = 1$, and $n_0(p > p_f) = 0$; The second one is FC with $0 < n_0(p_i < p < p_f) < 1$ and the single-particle spectrum $\varepsilon(p_i < p < p_f) = \mu$. The Fermi momentum p_F satisfies the condition $p_i < p_F < p_f$.

possesses its own set of zero-sound modes. The mode of interest for our analysis is that of transverse zero sound with its T-dependent sound velocity $c_t \simeq \sqrt{T/M}$ and the Debye temperature [29]

$$T_D \simeq c_t k_{max} \simeq \beta\sqrt{TT_F}. \tag{6}$$

Here, β is a factor, M is the effective mass of electron formed by vHs or by the hybridization, T_F is the Fermi temperature, while M^* is the effective mass formed finally by some interaction, e.g. the Coulomb interaction, generating flat bands [20]. The characteristic wave number k_{max} of the soft transverse zero-sound mode is estimated as $k_{max} \sim p_F$ since we assume that the main contribution forming the flat band comes from vHs or from the hybridization. We note that the numerical factor β cannot be established and is considered as a fitting parameter, rendering of T_D given by Eq. (6) correspondingly uncertain. Estimating $T_F \sim 10$ K and taking $\beta \sim 0.3$, and observing that the quasi-classical regime takes place at $T > T_D \simeq \beta\sqrt{TT_F}$, we obtain that $T_D \sim 1$ K and expect that strongly correlated Fermi systems can exhibit a quasi-classical behavior at their quantum criticality [29, 30] with the low-temperature coefficient A entering Eq. (1) $A = A_{LT}$. In case of HF metals with their few bands crossing Fermi level and populated by LFL quasiparticles and by HF quasiparticles, the transverse zero sound make the resistivity possesses the T-linear dependence at the quantum criticality as the normal sound (or phonons) do in the case of ordinary metals [30]. It is quite natural to assume that the sound scattering in these

materials is near material-independent, so that electron-phonon processes both in the low temperature limit at the quantum criticality and in the high temperature limit of ordinary metals have the same T-linear scattering rate that can be expressed as

$$\frac{1}{\tau T} \sim \frac{k_B}{\hbar}. \tag{7}$$

Thus, in case of the same material the coefficient $A = A_{HT}$, defining the classical linear T-dependence generated by the common sound (or phonons) at high temperatures, coincides with that of low-temperature coefficient A_{LT}, $A_{HT} \simeq A_{LT}$. As we shall see, this observation is in accordance with measurements on $Sr_3Ru_2O_7$ [1]. It is worth noting that the transverse zero sound contribution to the heat capacity C follows the Dulong-Petit law, making C possess a T-independent term C_0 at $T \gtrsim T_D$, as it does in case of ordinary metals [29]. It is obvious that the zero sound contributes to the heat transport as the normal sound does in case of ordinary metals, and its presence can violate the Wiedemann-Franz low; a detailed consideration of the emergence of zero sound and its properties will be published elsewhere.

There is another mechanism contributing to the T-linear dependence at the quantum criticality that we name the second mechanism in contrast to the first one described above and related to the transverse zero sound. We turn to consideration of the next contribution to the resistivity ρ in the range of quantum criticality, at which the dispersion of the flat band is governed by Eq. (5). It follows from Eq. (5) that the temperature dependence of $M^*(T)$ of the FC quasiparticles is given by

$$M^*(T) \sim \frac{\eta p_F^2}{4T}, \tag{8}$$

where $\eta = L/p_F$ [9–11]. Thus, the effective mass of FC quasiparticles diverges at low temperatures, while their group velocity, and hence their current, vanishes and the main contribution to the resistivity is provided by light quasiparticles bands. Nonetheless, the FC quasiparticles still play a key role in determining the behavior of both the T-dependent resistivity and ρ_0. The resistivity has the conventional dependence [2]

$$\rho(T) \propto M_L^* \gamma \tag{9}$$

on the effective mass and damping of the normal quasiparticles. Based on a fact that all the quasiparticles have the same lifetime, one can show that in playing its key role, the FC makes all quasiparticles of light and flat bands possess the same unique width γ and lifetime τ_q given by Eq. (2) [4, 31]. As a result, the first term a_0 on the right hand side of Eq. (2) forms an irregular residual resistivity ρ_0^c, while the second one forms the T-dependent part of the resistivity. The term "residual resistivity" ordinarily refers to impurity scattering. In the present case, the irregular residual resistivity ρ_0^c is instead determined by the onset of a flat band, and has no relation to scattering of quasiparticles by impurities [4]. The two mechanisms described above contribute to the coefficient A on the right hand side of Eq. (1) and it can be represented as $A \simeq A_{LT} + A_{FC}$, where A_{LT} and A_{FC} are formed by the zero sound and by FC, respectively. Coefficients A_{LT} and A_{FC} can be identified and differentiated experimentally for A_{LT} is accompanied by the temperature independent heat capacity C_0, while A_{FC} is escorted by the emergence of ρ_0^c.

We now consider the HF compound $Sr_3Ru_2O_7$ to illustrate the emergence of the both mechanisms contributing to the linear T-dependence of the resistivity. To achieve a connected picture of the quantum critical regime underlying the the quasi-classical region in $Sr_3Ru_2O_7$, we have to construct its $T-B$ phase diagram. We employ the model [21–28] based on vHs that induces a peak in the single-particle density of states (DOS) and leads a field-induced flat band [32]. At fields in the range $B_{c1} < B < B_{c2}$, the vHs is moved through the Fermi energy and the DOS peak turns out to be at or near the Fermi energy. A key point in this scenario is that within the range $B_{c1} < B < B_{c2}$, a repulsive interaction (e.g., Coulomb) is sufficient to induce FC and formation of a flat band with the corresponding DOS singularity locked to the Fermi energy [9–11, 32]. Now, it is seen from Eq. (5) that finite temperatures, while removing the degeneracy of the FC spectrum, do not change the excess entropy S_0, threatening the violation of the Nernst theorem. To avoid such an entropic singularity, the FC state must be altered as $T \to 0$, so that S_0 is to be removed before zero temperature is reached. This can take place by means of some phase transition or crossover, whose explicit consideration is beyond the scope of this paper. In case of $Sr_3Ru_2O_7$, this mechanism is naturally identified with the electronic nematic transition [21–23].

The schematic $T-B$ phase diagram of $Sr_3Ru_2O_7$ based on the proposed scenario is presented in Fig. 2. Its main feature is the magnetic field-induced quantum critical domain created by quantum critical points situated at B_{c1} and B_{c2}, generating FC and associated flat band. In contrast to the typical phase diagram of a HF metal [9], the domain occupied by the ordered phase in Fig. 2 is seen to be approximately symmetric with respect to the magnetic field $B_c \simeq (B_{c2} + B_{c1})/2 \simeq 7.9$ T [25]. The emergent FC and quantum critical points are considered to be hidden or concealed in a phase transition. The area occupied by this phase transition is indicated by horizontal lines and restricted by the thick boundary lines. At the critical temperature T_c where the new (ordered) phase sets in, the entropy is a continuous function. Therefore the top of the domain occupied by the new phase is a line of second-order phase transitions. As T is lowered, some temperatures $T_{\rm tr}^1$ and $T_{\rm tr}^2$ are reached at which the entropy of the ordered phase becomes larger than that of the adjacent disordered phase, due to the remnant entropy S_0 from the highly entropic flat-band state. Therefore, under the influence of the magnetic field, the system undergoes a first-order phase transition upon crossing a

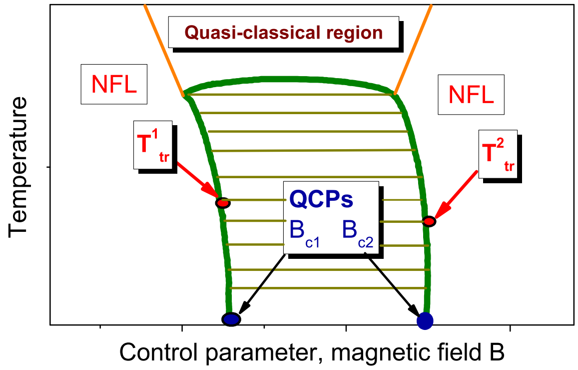

FIG. 2: (Color on line) Schematic phase diagram of the metal $Sr_3Ru_2O_7$. The quantum critical points situated at the critical magnetic fields B_{c1} and B_{c2} are indicated by arrows. The ordered phase bounded by the thick curve and demarcated by horizontal lines emerges to remove the entropy excess given by Eq. (4). Two arrows label the tricritical points $T^1_{\rm tr}$ and $T^2_{\rm tr}$ at which the lines of second-order phase transitions change to the first order. Quasi-classical region is confined by two lines at the top of the figure and by the top line of the ordered phase.

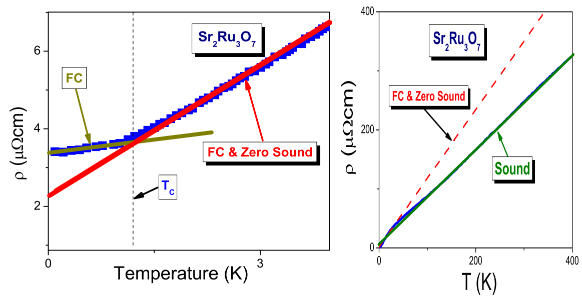

FIG. 3: (Color on line) The left panel: The resistivity $\rho(T)$ for $Sr_3Ru_2O_7$ at the critical field of $B_c = 7.9$ T [25]. Two straight lines display the T-linear dependence of the resistivity exhibiting a kink at $T = T_c$. At $T > T_c$ the T-linear resistivity is formed by the zero sound and FC contributions, while at $T < T_c$ the linear part of the resistivity comes from the FC contribution. The right panel: The resistivity at B_c over an extended temperature range up to 400 K [1]. The dash line shows the extrapolation of the low-T-linear resistivity at $T > T_c$, and the solid line shows the extrapolation of the high-T-linear resistivity formed at $T > 100$ K by the common sound.

sidewall boundary at $T = T^1_{\rm tr}$ or $T = T^2_{\rm tr}$, since entropy cannot be equalized there. It follows, then, that the line of second-order phase transitions is changed to lines of first-order transitions at tricritical points indicated by arrows in Fig. 2. It is seen from Fig. 2 that the sidewall boundary lines are not strictly vertical, due to the stated behavior of the entropy at the boundary and as a consequence of the magnetic Clausius-Clapeyron relation (as discussed in [23, 24]). Quasi-classical region is located above the top of the second order phase transition and restricted by two lines shown in Fig. 2. Therefore, the T-linear dependence is located in the same region and represented by AT with $A \simeq A_{LT} + A_{FC}$. We predict that in this region the heat capacity C contains the temperature independent term C_0 as that of the HF metal $YbRh_2Si_2$ does [33], while jumps of the residual resistivity, represented by ρ_0^c in $Sr_3Ru_2O_7$ [21], are generated by the second mechanism [4, 32].

The coefficients A_{FC}, A_{LT} and A_{HF} can be extracted from measurements of the resistivity $\rho(T)$ shown in the left and right panels of Fig. 3 [1, 25]. For the sake of clearness, the left panel shows only a part of the data on $\rho(T)$ that was measured from 0.1 K to 18 K at B_c, and exhibits the T-linear dependence between 1.4 K and 18 K and between 0.1 K and 1 K [25]. The coefficient $A \simeq A_{LT} + A_{FC} \simeq 1.1$ $\mu\Omega$cm/K between 18 K and 1.4 K. Since $T_D \sim 1$ K, we expect that between 1 K and 0.1 K the coefficient A is formed by the second mechanism and $A_{FC} \simeq 0.25$ $\mu\Omega$cm/K. The right panel reports the measurements of $\rho(T)$ for $T > T_c$ up to 400 K [1]. The dash line shows the extrapolation of the low-temperature linear resistivity at $T < 20$ K and B_c with $A \simeq 1.1$ $\mu\Omega$cm/K, and the solid line shows the extrapolation of the high-temperature linear resistivity at $T > 100$ K with $A_{HT} \simeq 0.8$ $\mu\Omega$cm/K [1]. The obtained values of A allow us to estimate the coefficients A_{LT} and A_{FC}. Due to our assumption that $A_{LT} \simeq A_{HT}$, we have $A - A_{LH} \simeq A_{FC} \simeq 0.3$ $\mu\Omega$cm/K, this value is in good agreement with $A_{FC} \simeq 0.25$ $\mu\Omega$cm/K. As a result, we conclude that for $Sr_3Ru_2O_7$ with its precise measurements the scattering rate is given by Eq. (7), and does not depend on T, provided that $T \geq T_D$. On the other hand, at $T < T_D$ $A_{HT}/A_{FC} \simeq 3$ and the constancy of the lifetime τ is violated, while the resistivity exhibits the T-linear dependence. It is seen from the left panel of Fig. 3, that the change from the resistivity characterized by the coefficient A_{LT} to the resistivity with A_{FC} is seen as a kink at $T_c = 1.2$ K representing both the entry into the ordered phase and a transition region at which the resistivity alters it slope. We expect that the constancy can also fail in such HF metals as $YbRh_2Si_2$ and the quasicrystal $Au_{51}Al_{34}Yb_{15}$ that exhibits the heavy-fermion behavior [34, 35].

In summary, we have shown that the quasi-classical physics describes the T-linear dependence of the resistivity of strongly correlated metals at their quantum criticality since flat bands, forming the quantum criticality, generate transverse zero-sound mode with the Debye temperature located within the quantum criticality area. Therefore, the T-linear dependence is formed by electron-phonon scattering in both ordinary metals and strongly correlated ones. As a result, we have demonstrated that

it is electron-phonon scattering that leads to the near material-independence of the lifetime τ that is expressed as $1/(\tau T) \sim k_B/\hbar$. We have shown that there can be another mechanism supporting the T-linear dependence even at $T < T_D$, which makes fail the constancy of τ regardless of the presence of T-linear dependence of the resistivity.

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